V E R I F I C A T I O N # 4

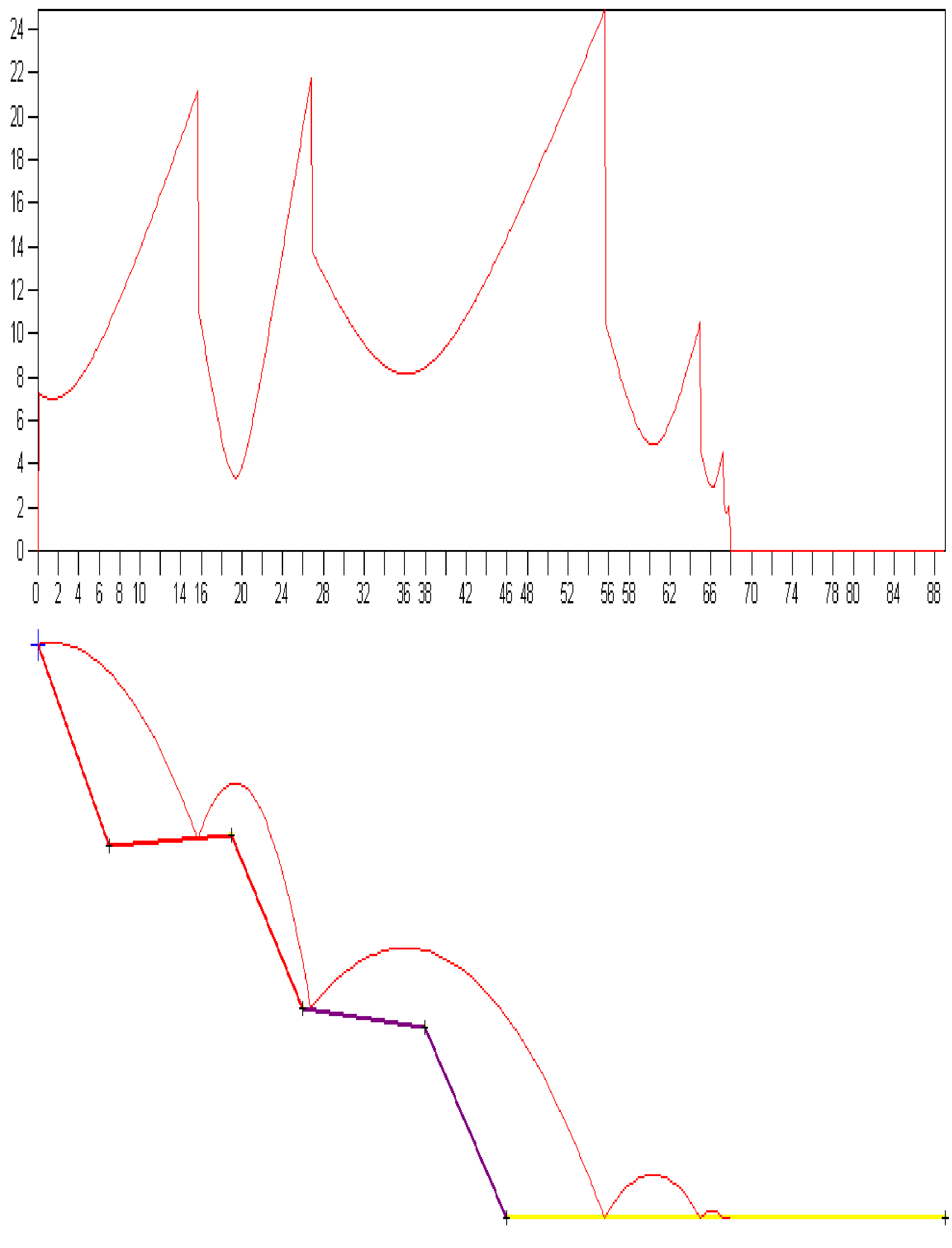
Envelopes

The purpose of this verification is to confirm that the envelopes produced by the program are correct and that the graphs based on these envelopes are being generated properly. There are three envelopes in the program: the kinetic energy envelope, the velocity envelope, and the bounce-height envelope. Each envelope is defined by the maximum value (e.g. maximum velocity) at a number of evenly spaced horizontal locations along the slope profile. The kinetic energy envelope measures the highest kinetic energy that any rock attained while passing each horizontal location. The velocity envelope measures the highest velocity that any rock attained while passing each horizontal location. The bounce-height graph measures the maximum height that any rock reached, minus the slope height, at each horizontal location (i.e. the maximum height *above* the slope). These envelopes are often used to determine where remedial measures should be placed, so it is very important that they operate correctly.

In the process of completing this verification, we will take advantage of the fact that the projectile verification has already been performed. The calculations that were performed in the projectile verification will provide the velocity of the rock at all of the locations that will be of interest in this verification. For example, the horizontal velocity, immediately before impact, (V_{XB}) for the first step of this verification is the same value as V_{XB} in the first step of the projectile verification.

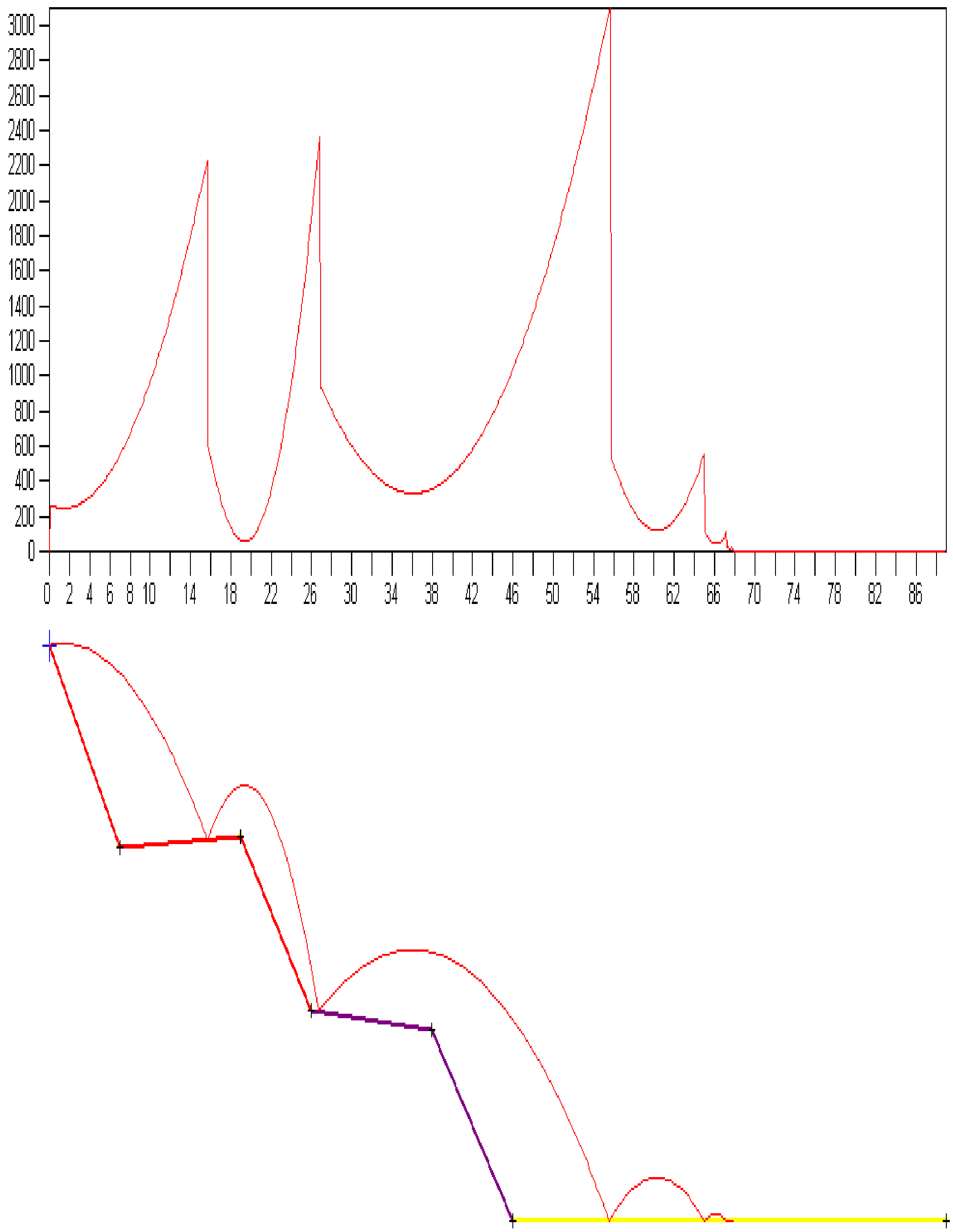
Velocity and Kinetic Energy Envelopes

We can see from inspection of Figures A.4.1 and A.4.2 that the velocity and kinetic energy envelopes are of the correct shape:



Maximum velocity [m/s] vs. distance along slope profile [m]

Figure A.4.1 - Velocity envelope



Maximum kinetic energy [J] vs. distance along slope profile [m]

Figure A.4.2 - Kinetic energy envelope

The velocity and kinetic energy envelopes have peaks and troughs at the correct locations and the curvature of each section appears to be correct. The discontinuities occur at the correct locations (where the rock impacts the slope, and looses energy). Since the shape of the graphs appeared to be correct, the graphs were verified by checking the value of each graph at significant locations along the slope profile. The values were checked at locations just before, and just after, each discontinuity. The values were also checked at the peak of each rock trajectory (the troughs on the velocity and kinetic energy graphs).

It is useful to note that at the peak of the rock's trajectory (the top of the parabola) the vertical velocity is zero. Since the horizontal velocity does not change while the rock is in the air, the only velocity that the rock possesses at the peak of the trajectory is its initial horizontal velocity. Therefore, the velocity at the peak of each parabolic path (the troughs of the velocity and kinetic energy envelopes) is equal to the post-impact velocity (V_{XA}) of the previous impact.

The same geometry and parameters were entered into RocFall and a simulation was performed. The "number of bins to use when plotting" option in RocFall was set to 1000. This value was chosen so that the program would collect data for the envelopes at numerous locations, especially those locations that are close to the points of interest.

The mass of the rock in the projectile verification was 10 kg. This mass is used to calculate the kinetic energy of the rock at each of the locations of interest.

Step 1

The velocity and kinetic energy just before the first impact (at x ≅ 15.7 m) are calculated: (The velocities are taken from the corresponding step of the projectile verification).

$$V_B = \sqrt{V_{XB}^2 + V_{YB}^2} = \sqrt{(7)^2 + (-20)^2} = 21.19\ m/s$$

$$KE_B = 0.5(m)V_B^2 = 0.5(10)(21.19)^2 = 2245\ J$$

The velocity and kinetic energy just after the first impact are calculated:

$$V_A = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(3.38)^2 + (10.59)^2} = 11.12\ m/s$$

$$KE_A = 0.5(m)V_A^2 = 0.5(10)(11.12)^2 = 618\ J$$

The velocity and kinetic energy at the second peak of the rock trajectory (at x ≅ 19.4 m) are calculated:

$$V_{PEAK} = V_{XA} = 3.38\ m/s$$

$$KE_{PEAK} = 0.5(m)V_{PEAK}^2 = 0.5(10)(3.38)^2 = 57.1\ J$$

A comparison of the results produced by manual calculation and by the program are presented in the following table:

	Hand Calculation	RocFall	Difference
V_B	21.19	21.14	0.24%
V_A	11.12	11.06	0.54%
V_{PEAK}	3.38	3.38	-
KE_B	2245	2234	0.49%
KE_A	618	612	0.97%
KE_{PEAK}	57.1	57.2	0.17%

Table A.4.1 - Comparison of velocity and kinetic energy results for step 1

Step 2

The velocity and kinetic energy just before the second impact (at x ≅ 26.8 m) are calculated:

$$V_B = \sqrt{V_{XB}^2 + V_{YB}^2} = \sqrt{(3.38)^2 + (-21.5)^2} = 21.76\ m/s$$

$$KE_B = 0.5(m)V_B^2 = 0.5(10)(21.76)^2 = 2367\ J$$

The velocity and kinetic energy just after the second impact are calculated:

$$V_A = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(8.14)^2 + (11.21)^2} = 13.85\ m/s$$

$$KE_A = 0.5(m)V_A^2 = 0.5(10)(13.85)^2 = 959\ J$$

The velocity and kinetic energy at the third peak of the rock trajectory (at x ≅ 36.1 m) are calculated:

$$V_{PEAK} = V_{XA} = 8.14\ m/s$$

$$KE_{PEAK} = 0.5(m)V_{PEAK}^2 = 0.5(10)(8.14)^2 = 331\ J$$

A comparison of the results produced by hand calculation and by the program are presented in the following table:

	Hand Calculation	RocFall	Difference
V_B	21.76	21.74	0.1%
V_A	13.85	13.78	0.5%
V_{PEAK}	8.14	8.14	-
KE_B	2367	2363	0.17%
KE_A	959	949	1.0%
KE_{PEAK}	331	331	-

Table A.4.2 - Comparison of velocity and kinetic energy results for step 2

Step 3

The velocity and kinetic energy just before the third impact (at x ≅ 55.6 m) are calculated:

$$V_B = \sqrt{V_{XB}^2 + V_{YB}^2} = \sqrt{(8.12)^2 + (-23.55)^2} = 24.91\ m/s$$

$$KE_B = 0.5(m)V_B^2 = 0.5(10)(24.91)^2 = 3103\ J$$

The velocity and kinetic energy just after the third impact are calculated:

$$V_A = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(4.88)^2 + (9.42)^2} = 10.61\ m/s$$

$$KE_A = 0.5(m)V_A^2 = 0.5(10)(10.61)^2 = 563\ J$$

The velocity and kinetic energy at the fourth peak of the rock trajectory (at x ≅ 60.3 m) are calculated:

$$V_{PEAK} = V_{XA} = 4.88\ m/s$$

$$KE_{PEAK} = 0.5(m)V_{PEAK}^2 = 0.5(10)(4.88)^2 = 119.1\ J$$

A comparison of the results produced by hand calculation and by the program are presented in the following table:

	Hand Calculation	RocFall	Difference
V_B	24.91	24.90	0.04%
V_A	10.61	10.48	1.23%
V_{PEAK}	4.88	4.88	-
KE_B	3103	3099	0.13%
KE_A	563	549	2.49%
KE_{PEAK}	119.1	119.2	0.08%

Table A.4.3 - Comparison of velocity and kinetic energy results for step 3

Step 4

The velocity and kinetic energy just before the fourth impact (at x ≅ 55.6 m) are calculated:

$$V_B = \sqrt{V_{XB}^2 + V_{YB}^2} = \sqrt{(4.88)^2 + (-9.42)^2} = 10.61\ m/s$$

$$KE_B = 0.5(m)V_B^2 = 0.5(10)(10.61)^2 = 563\ J$$

The velocity and kinetic energy just after the fourth impact are calculated:

$$V_A = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(2.93)^2 + (3.77)^2} = 4.77\ m/s$$

$$KE_A = 0.5(m)V_A^2 = 0.5(10)(4.77)^2 = 111\ J$$

The velocity and kinetic energy at the fifth peak of the rock trajectory (at x ≅ 60.3 m) are calculated:

$$V_{PEAK} = V_{XA} = 2.93\, m/s$$

$$KE_{PEAK} = 0.5(m)V_{PEAK}^2 = 0.5(10)(2.93)^2 = 42.9\ J$$

A comparison of the results produced by hand calculation and by the program are presented in the following table:

	Hand calculation	RocFall	Difference
V_B	10.61	10.52	0.85%
V_A	4.77	4.67	2.1%
V_{PEAK}	2.93	2.93	-
KE_B	563	553	1.78%
KE_A	111	109	1.80%
KE_{PEAK}	42.9	42.9	-

Table A.4.4 - Comparison of velocity and kinetic energy results for step 4

Bounce-Height Envelope

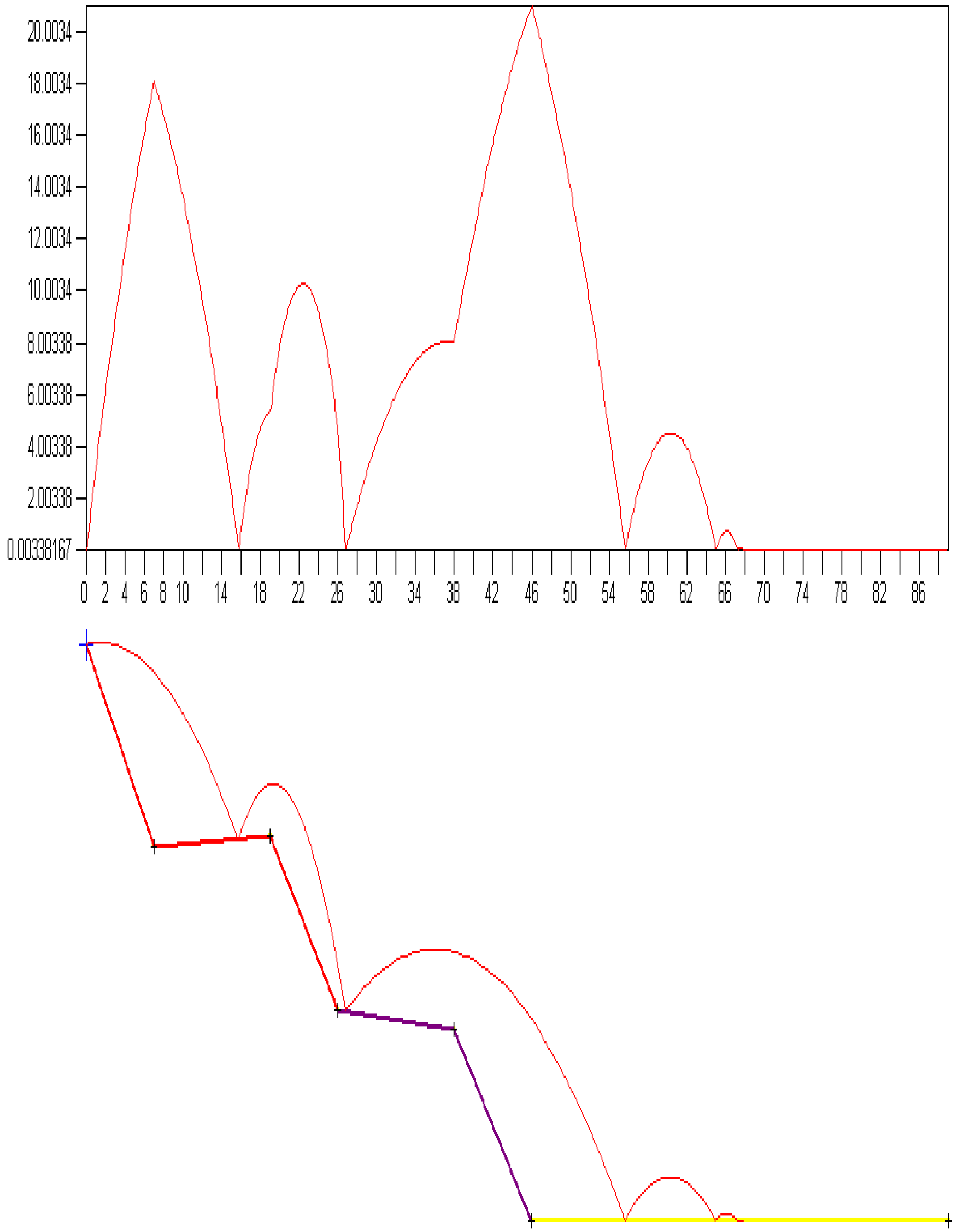
We can see by inspection of Figure A.4.3 that the bounce-height envelope is of the correct shape. The curvature of each section appears to be correct and the peaks occur at the correct locations. The bounce-height is also zero at the correct locations (where the rock impacts the slope). Since the shape of the bounce-height envelope appeared to be correct, the graph was verified by checking the value (the bounce-height) of the graph at significant locations along the slope profile. The value was checked at each of the slope vertices. The vertices offered a good place to check the values on the graph because they correspond to either an abrupt change in slope of the graph or a peak on the bounce-height envelope.

The bounce-height at the second slope vertex is calculated:

$$t = \frac{(X_2 - X_1)}{V_X} = \frac{(7-0)}{7} = 1\, s$$

$$y = Y_0 + V_Y t + 0.5gt^2 = 60 + 2(1) + 0.5(-9.81)(1)^2 = 57.1\, m$$

$$\Delta h = y - H_S = 57.1 - 39 = 18.100\ m$$



Maximum height above slope [m] vs. distance along slope profile [m]

Figure A.4.3 - Bounce-height envelope

The bounce-height at the third slope vertex is calculated:

$$t = \frac{(X_2 - X_1)}{V_X} = \frac{(19 - 15.732)}{3.38} = 0.967\ s$$

$$y = Y_0 + V_Y t + 0.5gt^2 = 39.728 + 10.59(0.967) + 0.5(-9.81)(0.967)^2 = 45.380\ m$$

$$\Delta h = y - H_S = 45.38 - 40 = 5.380\ m$$

The bounce-height at the fourth slope vertex is calculated:

$$t = \frac{(X_2 - X_1)}{V_X} = \frac{(26 - 15.732)}{3.38} = 3.04\ s$$

$$y = Y_0 + V_Y t + 0.5gt^2 = 39.728 + 10.59(3.04) + 0.5(-9.81)(3.04)^2 = 26.648\ m$$

$$\Delta h = y - H_S = 26.648 - 22 = 4.648\ m$$

The bounce-height at the fifth slope vertex is calculated:

$$t = \frac{(X_2 - X_1)}{V_X} = \frac{(38 - 26.8)}{8.14} = 1.38\ s$$

$$y = Y_0 + V_Y t + 0.5gt^2 = 21.867 + 11.21(1.38) + 0.5(-9.81)(1.38)^2 = 28.008\ m$$

$$\Delta h = y - H_S = 28.008 - 20 = 8.008\ m$$

The bounce-height at the sixth slope vertex is calculated:

$$t = \frac{(X_2 - X_1)}{V_X} = \frac{(46 - 26.8)}{8.14} = 2.36\ s$$

$$y = Y_0 + V_Y t + 0.5gt^2 = 21.867 + 11.21(2.36) + 0.5(-9.81)(2.36)^2 = 21.028\ m/s$$

$$\Delta h = y - H_S = 21.028 - 0 = 21.028\ m$$

A comparison of the results produced by hand calculation and by the program are presented in the following table:

Vertex	Hand Calculation	RocFall	Difference
2	18.1	18.06	0.22%
3	5.38	5.37	0.19%
4	4.648	4.719	1.50%
5	8.008	8.010	0.03%
6	21.028	20.999	0.14%

Table A.4.5 - Comparison of bounce-height results

Conclusion

The results were very similar in all cases. The difference between the manual calculations and the values produced by RocFall were less than 2.5 % in all cases, and typically much less. Therefore, it seems that the velocity envelope, the kinetic energy envelope, and the bounce-height envelope are all being produced correctly.

The reason the results produced by RocFall do not correspond *exactly* to the manual calculations is because the points on the envelope are not at exactly the same locations as the points used for the manual calculations. The program collects data for the envelopes at a number locations evenly spaced along the slope profile (the number of locations used in this example was 1000). Since the program only collects data at these points there will be many locations where there is no data (i.e. in the spaces between the envelope points). In these cases, the envelope value at the closest horizontal location was used. For example: The value for the bounce-height, by manual calculation, at x = 46.000 m was 21.028 m. The program produced results at x = 45.924 m (bounce-height = 20.929 m) and x = 46.013 m (bounce-height = 20.999 m). Since the peak value is at 46.000 m and the two locations produced by the program border this location they will be slightly lower, and the results will not be exact.

Increasing the number of locations used to collect data would have decreased the difference between the program results and the manual calculations. However, given the lack of certainty in much of the input data (e.g. R_T), the additional precision is of questionable value.